Int J Theor Phys (2009) 48: 1177–1185
DOI 10.1007/s10773-008-9891-y

Anisotropic Bulk Viscous Cosmological Models with Variable G and Λ

G.P. Singh · A.Y. Kale

Received: 31 August 2008 / Accepted: 29 October 2008 / Published online: 6 November 2008

Abstract This paper deals with Bianchi-I, Kantowski Sachs and Bianchi-III anisotropic cosmological models of the universe, filled with a bulk viscous cosmic fluid, in the presence of variable gravitational and cosmological constants. A new set of exact solutions of Einstein's field equation have been obtained in both truncated and full causal theories. Physical behaviour of the models has also been discussed.

Keywords Anisotropy · Bulk viscosity · Gravitational and cosmological 'constants'

1 Introduction

The early stages of the universe did not have the property of isotropy as we find it today. Astronomical and astrophysical observations reveals that during evolution, the universe is isotropized and on a very large scale (>100 Mps) it is homogeneous and isotropic. It is well known that the relativistic cosmological models for spatially homogeneous and anisotropic space-time belong either to the Bianchi types or Kantowaski-Sachs space-time. These Anisotropic Cosmological models have been extensively studied in the literature by a number of authors ([1–11] etc.) in different contexts.

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. To study the effect of the bulk viscosity the first relativistic theory of non-equilibrium thermodynamics was developed by Eckart [12] and subsequent study by several authors viz. Muller [13], Israel and Stewart [14], Hiscock and Lindblon [15]

G.P. Singh (✉)
Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur 440010, India
e-mail: gpsingh@mth.vnit.ac.in

A.Y. Kale
Department of Mathematics, St.Vincent Pallotti College of Engineering and Technology, Nagpur, India
e-mail: ashwini_kale@rediffmail.com

reveals that the Eckart type theories suffer from serious drawbacks concerning causality and stability. Gron [16] and Maartens [17] have presented exhaustive review of non-causal and causal viscous expanding model of the universe respectively. Several authors viz. Zimdhal [18], Arbab [19], Singh et al. [20], Singh and Beesham [21], Bali and Dave [22, 23], Mak and Harko [24], Wang [25], Yadav et al. [26], Calistete et al. [27] and others have discussed a variety of bulk viscous isotropic and anisotropic cosmological models.

In the last more than one decade the remarkable progress made in various types of astrophysical and cosmological observations has profoundly changed the cosmology by predicting an accelerated expansion of the universe. The existence of a positive cosmological 'constant' is strongly favoured by Super-novae(SNe)Ia observations and anisotropy measurements of cosmic microwave background (CMB) made by the WMAP experiment [28–36]. It has been suggested in the literature that a dynamical cosmological 'constant' (Λ) is important to accommodate an accelerating universe. Further, it should be considered as a function of time, so that it was large in the early universe and relaxed to small with expansion of the universe [37–44] and references therein). Thus, the Λ which has fallen in and out of researcher's interest several times, remains a focal point of research interest in modern cosmological theories due to its ability to resolve many outstanding cosmological problems in natural way.

The cosmological constant Λ and the gravitational constant G are the two parameters present in Einstein's field equations. The Newtonian constant of gravitation, G, plays the role of coupling constant between geometry and matter in Einstein field equations. Dirac [45] was the first researcher, to propose the idea of a variable G on certain physical grounds. In the last few decades there have been numerous modifications of general relativity in which gravitational 'constant' G varies with time. In order to achieve possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity, many other extension of Einstein theory with time dependent G have also been proposed [46, 47]. Considering the principle of absolute quark confinement, Der Sarkissian [48] has suggested that gravitational and cosmological constant may be considered as function of time parameter in Einstein's theory of relativity. A number of authors Kalligas et al. [49], Arbab [19], Abdussattar and Vishwakarma [50], proposed linking of variation of G with Λ, within the framework of general relativity. This new approach is appealing since it leaves the form of Einstein equations formally unchanged by allowing a variation of G to be accompanied by change in Λ. The detailed analysis of FRW universes in a wide range of scalar-tensor theories of gravity has been performed by Barrow and Parsons [51]. Singh and Kotambkar [52] have discussed cosmological models with G and Λ in higher dimensional space time. Very recently, Singh and Sorokhaibam [53] have studied FRW cosmological models with the gravitational and cosmological constants and Singh et al. [54] have discussed Bianchi type I cosmological models with variable G and Λ term in general relativity. Chakraborty and Roy [55] have presented anisotropic cosmological models with bulk viscosity in the presence of variable G and Λ. Considering the aforesaid studies it is worthwhile to study anisotropic viscous cosmological models with variable G and Λ.

2 The Metric and Field Equations

The general metric for axially symmetric Bianchi I, Kantowski-Sachs and Bianchi III space time can be described by

$$\begin{aligned}
&ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)d\Omega_k^2,\\
&d\Omega_k^2 = dy^2 + dz^2 \quad \text{for } k=0 \quad \text{(Bianchi-I Model)},\\
&d\Omega_k^2 = d\theta^2 + \sin^2\theta d\phi^2 \quad \text{for } k=1 \quad \text{(Kantowski-Sachs Model)},\\
&d\Omega_k^2 = d\theta^2 + \sinh^2\theta d\phi^2 \quad \text{for } k=-1 \quad \text{(Bianchi-III Model)}.
\end{aligned} \tag{1}$$

Einstein field equations with cosmological 'constant' is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} + \Lambda g_{ij}, \tag{2}$$

where R_{ij} is the Ricci Tensor and T_{ij} is the energy momentum tensor of cosmic fluid in the presence of bulk viscosity defined as

$$T_{ij} = (\rho + p + \Pi)u_i u_j - (p + \Pi)g_{ij}. \tag{3}$$

Here ρ is the energy density, p represents equilibrium pressure and Π stands for bulk viscous stress respectively and $u_i$is the flow vector satisfying the relation $u^i u_i = 1$.

The Einstein's field (2) for the space-time metric (1) yield following equations

$$2\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot b^2}{b^2} + \frac{k}{b^2} = 8\pi G\rho + \Lambda, \tag{4}$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{b} + \frac{\dot a}{a}\frac{\dot b}{b} = -8\pi G(p + \Pi) + \Lambda, \tag{5}$$

$$2\frac{\ddot b}{b} + \frac{\dot b^2}{b^2} + \frac{k}{b^2} = -8\pi G(p + \Pi) + \Lambda. \tag{6}$$

By combining (4)–(6) one can easily obtain continuity equation as

$$\dot\rho + (\rho + p + \Pi)\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) + \rho\frac{\dot G}{G} + \frac{\dot\Lambda}{8\pi G} = 0. \tag{7}$$

The energy-momentum conservation equation ($T^{ij}_{;j} = 0$) suggests

$$\dot\rho + (\rho + p + \Pi)\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) = 0. \tag{8}$$

From (7) and (8), we have

$$\rho\frac{\dot G}{G} + \frac{\dot\Lambda}{8\pi G} = 0. \tag{9}$$

For the full causal non equilibrium thermodynamics the causal evolution equation for bulk viscosity is

$$\Pi + \tau\dot\Pi = -\xi\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) - \frac{\varepsilon\tau\Pi}{2}\left(\frac{\dot a}{a} + 2\frac{\dot b}{b} + \frac{\dot\tau}{\tau} - \frac{\dot\xi}{\xi} - \frac{\dot T}{T}\right). \tag{10}$$

Here $T \geq 0$ is the absolute temperature, ξ is the bulk viscosity coefficient which cannot become negative otherwise the principle of entropy increase would be violated and the coefficient τ denotes the relaxation time for transient bulk viscous effects. Causality requires $\tau > 0$. When $\varepsilon = 0$, (10) reduces to evolution equation for truncated theory. For full causal theory $\varepsilon = 1$ and the non-causal theory (Eckart's theory) has $\tau = 0$.

3 Cosmological Solutions

It can be easily seen that we have three independent equations having seven unknowns a, b, ρ, p, Π, G and Λ. In order to obtain exact solution, we require four more physically plausible relations amongst the variables. As majority of cosmological models belong to either power law form or exponential form, we presently considering a power law relation between scale factors and time coordinate along with well known relations for pressure, density and cosmological 'constant' as

(i) Power law relation for scale factors

$$a = a_0 t^{\alpha}, \qquad b = b_0 t^{\beta} \tag{11}$$

(ii) Barotropic equation of state

$$p = \gamma\rho, \quad 0 \le \gamma \le 1 \tag{12}$$

(iii) The variation of Λ [19]

$$\Lambda = \Lambda_0 \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)^2, \tag{13}$$

where $a_{0,} b_0$, α, β, Λ_0 are arbitrary constants.

Since the right hand sides of (5) and (6) are identical, we have

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = 2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{k}{b^2}. \tag{14}$$

Which gives $\alpha = \beta$ or $\alpha + 2\beta = 1$ for $k = 0$ and $\alpha^2 - 1 = \frac{k}{b_0^2}$ for $k \neq 0$.

Case I In this case considering $k = 0$ and $\alpha = \beta$ from (11), we have

$$a = a_0 t^{\alpha}, \qquad b = b_0 t^{\alpha}. \tag{15}$$

The CMBR and supernova observation suggest $0.85 \le H_0 t_0 \le 1.13$. That gives $0.85 \le \alpha \le 1.13$. Using these conditions the set of (4)–(6), (9) and (13) yield

$$\rho = \rho_0 t^{-(m+2)}, \tag{16}$$

$$\Pi = -\Pi_0 t^{-(m+2)}, \tag{17}$$

$$\Lambda = \frac{9\Lambda_0 \alpha^2}{t^2}, \tag{18}$$

$$G = G_0 t^m. \tag{19}$$

Here Λ_0, G_0 are constants of integration and other constants have following relations

$$m = \frac{6\Lambda_0}{1 - 3\Lambda_0},$$

$$\rho_0 = \frac{3\alpha^2}{8\pi G_0}(1 - 3\Lambda_0),$$

$$\Pi_0 = \frac{\alpha}{8\pi G_0}\big[2 - 3\alpha(1 - 3\Lambda_0)(1 + \gamma)\big].$$

Using observational values of $\frac{\dot{G}}{G} \approx 10^{-11}$ [56] and $t_{\text{now}} \sim 10^{10}$, (19) suggests that $m = 0.1$.

In this case the deceleration parameter $q=-1-\frac{\dot{H}}{H^2}$, takes value -0.1150 which is well within observational limits.

(i) *Evaluation of Bulk Viscosity in Truncated Causal Theory.*

Now, we are interested in study of the variation in bulk viscosity coefficient (ξ) and relaxation time (τ) with respect to cosmic time. It has already been mentioned that for truncated theory $\varepsilon=0$ and hence (10) reduces to

$$\Pi+\tau\dot{\Pi}=-3\xi H. \tag{20}$$

Further, in order to have exact solutions of the system of equations one more physical plausible relation is required. Thus, we consider the well accepted relation

$$\tau=\frac{\xi}{\rho}. \tag{21}$$

With the help of (15)–(17), (19) and (21), we get

$$\xi=\xi_0\frac{1}{t^{(m+1)}}, \tag{22}$$

where

$$\xi_0=\frac{\Pi_0\rho_0}{[3\alpha\rho_0+(m+2)\Pi_0]}.$$

From (16), (21) and (22), it can be easily seen that τ has linear relation with cosmic time t.

(ii) *Evaluation of Bulk Viscosity in Full Causal Theory*

Here we consider model under full causal theory i.e. substituting $\varepsilon=1$ in (10) yield the evolution equation for full causal theory. Further, on the basis of Gibb's integrability condition, Maartens (1995) has suggested the equation of state for temperature as

$$T\propto \exp\int\frac{dp}{\rho+p}T,$$

which with the help of (12) gives

$$T=T_0\rho^{\frac{\gamma}{(1+\gamma)}}, \tag{23}$$

where T_0 stands for a constant. With the help of (16) and (23) one can easily obtain an expression for temperature in terms of cosmic time t as

$$T=T_0\rho_0^{\frac{\gamma}{(1+\gamma)}}\left(\frac{1}{t}\right)^{\frac{(m+2)\gamma}{(1+\gamma)}}. \tag{24}$$

Considering age of the universe 10^{10} years $\sim 3\times10^{17}$ sec, $\gamma=1/3$ and $m=1/10$, (24) suggests temperature for radiation dominated model $T\sim 1$ K which is in fair agreement with observed value $T\sim 3$ K.

From (10), (21) and (23), we get

$$\Pi+\frac{\xi}{\rho}\dot{\Pi}=-\xi\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)-\frac{\xi\Pi}{2\rho}\left[\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)-\frac{(1+2\gamma)}{(1+\gamma)}\frac{\dot{\rho}}{\rho}\right]. \tag{25}$$

Further, by use of (15)–(17) one can obtain the relation between bulk viscosity coefficient and cosmic time t as

$$\xi = \xi_1 \frac{1}{t^{(m+1)}}, \tag{26}$$

where the constant ξ_1 is defined as

$$\xi_1 = \frac{2\Pi_0\rho_0}{[2(m+2)\Pi_0 6\alpha\rho_0 - 3\alpha\Pi_0 - \frac{(1+2\gamma)}{(1+\gamma)}(m+2)\Pi_0]}.$$

It can be easily seen that always $\xi > 0$ and decreasing with evolution of the universe.

Case II In this case for further study, we consider $k = 0$ and $\alpha + 2\beta = 1$. Under this relation, (11) gives

$$a = a_0 t^{\alpha}, \qquad b = b_0 t^{\frac{1-\alpha}{2}}. \tag{27}$$

Using these conditions the set of (4)–(6), (9) and (13) reduce to

$$\rho = \rho_1 t^{-(r+2)}, \tag{28}$$

$$\Pi = -\Pi_1 t^{-(r+2)}, \tag{29}$$

$$\Lambda = \frac{9\Lambda_0}{t^2}, \tag{30}$$

$$G = G_0 t^r. \tag{31}$$

Here constants take the form

$$r = \frac{2\Lambda_0}{2\alpha + 1 - 3\alpha^2 - 4\Lambda_0},$$

$$\rho_1 = \frac{2\alpha - 3\alpha^2 + 1 - 4\Lambda_0}{32\pi G_0},$$

$$\Pi_1 = \frac{(1-\gamma)(3\alpha^2 - 2\alpha - 1) - 4\Lambda_0(1+\gamma)}{32\pi G_0}.$$

Again, using observational values of $\frac{\dot{G}}{G} \approx 10^{-11}$ and $t_{\text{now}} \sim 10^{10}$, (31) suggests that $r = 0.1$.

In this model the value of deceleration parameter is positive which shows decelerating behaviour of the cosmological model. It is worthwhile to mention the work of Vishwakarma [57] where he has shown that the decelerating model is also consistent with recent CMB observations made by WMAP, as well as with the high redshift supernovae Ia data including SN 1997ff at $z = 1.755$. Our further investigation in this direction may throw light on this ambiguity in future.

(i) *Evaluation of Bulk Viscosity in Truncated Causal Theory*

With the help of (20), (21) and (27)–(29), we obtain

$$\xi = \xi_2 \frac{1}{t^{(r+1)}}, \tag{32}$$

where

$$\xi_2 = \frac{\Pi_1\rho_1}{[\rho_1 + (r+2)\Pi_1]}.$$

This shows that bulk viscosity coefficient is decreasing with evolution of the universe.

(ii) *Evaluation of Bulk Viscosity in Full Causal Theory*

In this case, we consider full causal theory i.e. substituting $\varepsilon = 1$ in (10).

With the help of (23) and (28) the expression for temperature in terms of cosmic time t as

$$T = T_0 \rho_0^{\frac{\gamma}{(1+\gamma)}} \left(\frac{1}{t}\right)^{\frac{(r+2)\gamma}{(1+\gamma)}}. \tag{33}$$

In this case also the numerical value of temperature can be calculated similar to the previous case and it is within the observational limits.

With help of (25), (27)–(29), we get

$$\xi = \xi_3 \frac{1}{t^{(r+1)}}, \tag{34}$$

where

$$\xi_3 = \frac{2\Pi_1 \rho_1}{[2(r+2)\Pi_1 + 2\rho_1 - \frac{(1+2\gamma)}{(1+\gamma)}(r+2)\Pi_1]}.$$

Case III This case is dealing with Kantowski-Sachs and Bianchi type III cosmological models having $k \neq 0$, $\beta = 1$, $\alpha^2 - 1 = \frac{k}{b_0^2}$. Under these assumptions scale factors take the form

$$a = a_0 t^{\alpha}, \qquad b = b_0 t. \tag{35}$$

The CMBR and supernova observation suggest $0.85 \leq H_0 t_0 \leq 1.13$. That gives $0.55 \leq \alpha \leq 1.39$.

Using these conditions the set of (4)–(6), (9) and (13) reduces to

$$\rho = \rho_0 t^{-(n+2)}, \tag{36}$$

$$\Pi = -\Pi_0 t^{-(n+2)}, \tag{37}$$

$$\Lambda = \frac{9\Lambda_0(\alpha+2)^2}{t^2}, \tag{38}$$

$$G = G_0 t^n. \tag{39}$$

Here constants n, ρ_0 and Π_0 are related to other constants as

$$n = \frac{2\Lambda_0(\alpha+2)}{\alpha - \Lambda_0(\alpha+2)},$$

$$\rho_0 = \frac{\alpha^2 + 2\alpha - \Lambda_0(\alpha+2)^2}{8\pi G_0},$$

$$\Pi_0 = \frac{\alpha^2 + \gamma(\alpha^2+2\alpha) - \Lambda_0(\alpha+2)^2(1+\gamma)}{8\pi G_0}.$$

Further, the deceleration parameter takes value -0.1150 within observational limits.

(i) *Evaluation of Bulk viscosity in Truncated Causal Theory*

In the present case using the values of scale factor, energy density, viscous pressure from (35)–(37), (20)–(21) suggest

$$\xi = \xi_4 \frac{1}{t^{(n+1)}}, \tag{40}$$

where

$$\xi_4 = \frac{\Pi_0 \rho_0}{[\rho_0(\alpha+2) + (n+2)\Pi_0]}.$$

(ii) *Evaluation of Bulk Viscosity in Full Causal Theory*

In this case, we consider full causal theory i.e. substituting $\varepsilon = 1$ in (10).

With the help of (23) and (36) the expression for temperature in terms of cosmic time t as

$$T = T_0 \rho_0^{\frac{\gamma}{(1+\gamma)}} \left(\frac{1}{t}\right)^{\frac{(n+2)\gamma}{(1+\gamma)}}. \tag{41}$$

By use of (25), (35)–(37), we have obtained the bulk viscosity coefficient as

$$\xi = \xi_5 \frac{1}{t^{(n+1)}}, \tag{42}$$

where

$$\xi_5 = \frac{2\Pi_0 \rho_0}{[2(n+2)\Pi_0 + 2\rho_0(\alpha+2) - \Pi_0(\alpha+2) - \frac{(1+2\gamma)}{(1+\gamma)}(n+2)\Pi_0]}.$$

4 Discussion

In this paper we have studied axially symmetric Bianchi-I, Kantowski-Sachs and Bianchi-III space time models with bulk viscosity, cosmological and gravitational constants. In Power law form for $k = 0$ two cases $\alpha = \beta$ and $\alpha + 2\beta = 1$ have been discussed. When $\alpha = \beta$ we get accelerated expanding cosmological model for all $\alpha > 1$ whereas for $\alpha + 2\beta = 1$ we get $q = 2$. Though present day observations and literature favour accelerating model of the universe, the decelerating model should not be simply ruled out as our knowledge and instruments to measure various parameter of the universe are not complete and perfect. In the literature there are examples of some results which were in and out of scientific consideration. In both cases $k = 1$ and $k = -1$ cosmological models presented in case III show the accelerating model of the universe for all values of $\alpha > 1$. In all the cases energy density, pressure, bulk viscous stress and bulk viscosity coefficient are decreasing with evolution of the universe. The temperature predicted by our models is also within observational limits.

Considering the exponential form of the scale factors viz., $a = a_0 e^{\alpha t}$, $b = b_0 e^{\beta t}$, it can be easily seen that all parameters ρ, p, Π, G and Λ takes uniform values for all time which are not consistent with observational results. Hence exact solutions in this case are not presented.

Acknowledgements Authors would like to thank Inter-University Centre for Astronomy and Astrophysics, Pune for providing facilities where part of this work was completed.

References

1. Lorenz, D.: J. Phys. A **15**, 2997 (1982)
2. Huang, W.: J. Math. Phys. **31**, 1456 (1990)
3. Singh, T., Singh, G.P.: J. Math. Phys. **32**, 2456 (1991)
4. Singh, T., Singh, G.P.: Int. J. Theor. Phys. **31**, 1433 (1992)

5. Singh, T., Agrawal, A.K.: Int. J. Theor. Phys. **32**, 1041 (1993)
6. Pradhan, A., Singh, S.K.: Int. J. Mod. Phys. D **13**, 503 (2004)
7. Pradhan, A., Pandey, P., Singh, G.P., Deshpande, R.V.: Spacetime Subst. **6**, 116 (2005)
8. Saha, B.: Int. J. Theor. Phys. **45**, 952 (2005)
9. Saha, B.: Astrophys. Space Sci. **302**, 83 (2006)
10. Kumar, S., Singh, C.P.: Astrophys. Space Sci. **312**, 57 (2007)
11. Reddy, D.R.K., Naidu, R.L.: Astropyhs. Space Sci. **312**, 99 (2007)
12. Eckart, C.: Phys. Rev. **58**, 919 (1940)
13. Muller, I.: Z. Phys. **198**, 329 (1967)
14. Israel, W., Stewart, J.M.: Phys. Let. A. **58**, 213 (1976)
15. Hiscock, W.A., Lindblom, L.: Phys. Rev. D **35**, 3723 (1987)
16. Gron, O.: Astrophys. Space Sci. **173**, 191 (1990)
17. Maartens, R.: Class. Quantum Gravit. **12**, 455 (1995)
18. Zimdhal, W.: Phys. Rev. D. **53**, 5483 (1996)
19. Arbab, A.I.: Gen. Relativ. Gravit. **29**, 61 (1997)
20. Singh, T., Beesham, A., Mbokazi, W.S.: Gen. Relativ. Gravit. **30**, 573 (1998)
21. Singh, G.P., Beesham, A.: Aust. J. Phys. **52**, 1039 (1999)
22. Bali, R., Dave, S.: Astrophys. Space Sci. **282**, 461 (2002)
23. Bali, R., Dave, S.: Astrophys. Space Sci. **288**, 503 (2003)
24. Mak, M.K., Harko, T.: Int. J. Mod. Phys. D **11**, 447 (2002)
25. Wang, X.X.: Astrophys. Space Sci. **93**, 433 (2004)
26. Yadav, M.K., Roy, A., Pradhan, A.: Int J. Theor. Phys. **46**, 2677 (2007)
27. Calistete, Jr. R., Fabris, J.C., Tossa, J., Zimdhal, W.: Phys. Rev. D **76**, 103516 (2007)
28. Perlmutter, S.: Nature **391**, 51 (1998)
29. Perlmutter, S.: Astrophys. J. **517**, 565 (1999)
30. Riess, A.G., et al.: Astrophys. J. **116**, 1009 (1998)
31. Riess, A.G., et al.: Astrophys. J. **560**, 49 (2001)
32. Riess, A.G., et al.: Astrophys. J. **607**, 665 (2004)
33. Bennett, C.L., et al.: Astrophys. J. **583**, 1 (2003)
34. Hinshaw, G., et al.: Astrophys. J. Suppl. **170**, 288 (2007)
35. Hinshaw, G., et al.: astr-ph0732v1 (2008)
36. Spergel, D.N., et al.: Astrophys. J. Suppl. **170**, 377 (2007)
37. Weinberg, S.: Rev. Mod. Phys. **611** (1989)
38. Chen, W., Wu, V.S.: Phys. Rev. D. **41**, 695 (1990)
39. Abdel Rahman, A.M.: Phys. Rev. D **45**, 3492 (1992)
40. Ozer, M., Taha, M.O.: Phys. Lett. A **13**, 571 (1998)
41. Sahani, V., Starobinsky, A.: Int. J. Mod. Phys. D **9**, 373 (2000)
42. Vishwakarma, R.G.: Gen. Relativ. Gravit. **33**, 1973 (2001)
43. Vishwakarma, R.G.: Class. Quantum Gravit. **19**, 776 (2002)
44. Debnath, P.S., Paul, B.C.: Phys. Rev. D **76**, 123505 (2007)
45. Dirac, P.A.M.: Nature **130**, 323 (1937)
46. Brans, C., Dicke, R.H.: Phys. Rev. **24**, 925–936 (1961)
47. Barber, G.A.: Gen. Relativ. Gravit. **14**, 117 (1982)
48. Der Sarkissian, M.: Nuovo Cim. B **88**, 29 (1985)
49. Kalligas, D., Wesson, P., Everitt, C.W.F.: Gen. Relativ. Gravit. **24**, 351 (1992)
50. Abdussattar, Vishwakarma, R.G.: Class. Quantum Gravit. **14**, 945 (1997)
51. Barrow, J.D., Parsons, P.: Phys. Rev. D **55**, 1906 (1997)
52. Singh, G.P., Kotambkar, S.: Gravit. Cosmol. **9**, 206 (2003)
53. Singh, N.I., Sorokhaibam, A.: Astrophys. Space Sci. **310**, 131 (2007)
54. Singh, J.P., Pradhan, A., Singh, A.K.: Astrophys. Space Sci. **314**, 83 (2008)
55. Chakraborty, S., Roy, A.: Astrophys. Space Sci. **313**, 389 (2008)
56. Garcia-Berro, E., Kubyshin, Y., Loren-Aguilar, P., Isern, J.: Int. J. Mod. Phys. D **15**, 1163 (2006)
57. Vishwakarma, R.G.: Mon. Not. R. Astron. Soc. **345**, 545 (2003)